A Tale of Two CD'S

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These are indeed exciting times in the world of Mathematics. Stirred by the NCTM *Standards*, the winds of change are blowing through every level of the K-12 curriculum. At the same time, in colleges and universities the finest minds of our profession are turning their attention to the forging of a Calculus for a New Century. Earlier this year, we even saw the unexpected verification of Fermat's Last Theorem, something which surely none of us thought we would see in this lifetime. There is so much to talk about when mathematicians get together these days, yet, because we are teachers, we have so little time for talking. That is why I have chosen to write this article about none of these things.

Instead, I would like to write about records.

I have collected records since I was in grade school. By the time I was in graduate school, I owned more than two thousand 45's, three hundred albums, and miscellaneous 78's and EP's. I shared my hobby with friends, including some memorable years as music director and station manager of the college radio station at Holy Cross. I was one of those guys who could, upon hearing a golden oldie on the radio, quote the title, artist, and year of the song, and quite often the label and songwriter as well. My interest in the music of the moment naturally declined about the time disco music became popular, but by that time I had accumulated enough vinyl classics to keep myself and my party guests entertained forever. For example, one of my favorite ways to pass an evening with friends is to stage a "nostalgia playoff" between two guests, playing alternately the hit songs from their respective high school graduation years, until the quality of one year's hits is clearly unable to keep up with the quality of the other's. (In case you are interested, no guest has ever gone up against a graduate of 1957 without conceding after the 15th round or so.)

As a student of the recording industry, I would always sit up and take notice when some new product emerged which the prophets predicted would change the way people listened to music. The first big pretender to the vinyl throne was the 8-track tape. "The 8-track tape," they predicted, "will redefine the recording industry." It required no threading onto a spool, it did not scratch or shatter, it did not collect dust, it required no needle, it produced high fidelity sound for multiple speakers, and *you could play it in your car*! I was momentarily impressed, but I continued to buy records, and so did apparently a lot of other people. Today if you want to buy an 8-track tape you have to go to an antique show.

Then came the cassette tape. "The cassette tape," they predicted, "will refine the recording industry." It was smaller than the clumsy 8-track, but it had all its same advantages, including that of being playable in your automobile. Moreover, you could actually stick a few into your glove compartment. You could also buy a "portable tape player" which would play your cassettes on an arbitrary street corner at an arbitrary volume level. I myself continued to buy records, although I

did eventually buy a cassette recorder so that I could tape my records at home for playing later in my car. Although record stores eventually began selling albums on cassette, it was usually from a shelf toward the back of the store. Vinyl was still king.

Next came the laser disc. "The laser disc," they predicted, "will redefine the recording industry." The laser disc had the music encoded digitally, promising virtually perfect fidelity forever. It also would not scratch, smudge, or collect dust, and in place of the old diamond needle, which everyone always suspected would be fatal to plastic records eventually, there was a neat, powerful laser beam to lift the music off of the disc as cleanly as Scottie might beam up Captain Kirk. This was dazzling technology indeed; unfortunately, you could not play a laser disc in your car. For one thing, each disc was the size of a medium pizza; for another thing, you had to *sell* your car in order to buy a laser disc *player*, which cost several thousand dollars. Needless to say Hugh Hefner bought one for every room of his mansion, while the rest of us just kept buying records.

Then along came the compact disc. "The compact disc," they predicted, "will redefine the recording industry." And in an incredibly short period of time, it did.

Walk into a record store today and it probably won't even be *called* a record store. The bins that once stored the vinyl now store row upon row of CD's. Oh, the biggest stores will still carry a few records, but you often have to walk past the cassette section in order to find them. If you want a *real* adventure, try replacing your old phonograph needle! The recording industry has been completely taken over by the compact disc.

Why did the CD succeed where the other technologies had failed? Simply put, it was such a perfect idea that nobody who loved music could resist it. The sound was virtually perfect; the discs themselves were rugged; the players were affordable; and these things were, as their name implied, *compact*—you could fit dozens of them into a shoebox and carry them to the home of your friend. There, you would almost assuredly find another CD player. Everyone could share in the miracle.

Of course, for a while I held out. After all, I had this enormous investment in records, not to mention the means for playing them. But I would go the homes of my friends and hear the crystalline strains of CD music, and I would be jealous of that incredible *sound*. Finally, I realized that it was not the *records* that I liked; it was the *music*, and the music could be heard better on CD's. I bought myself a player and began collecting compact discs. Most of my first compact disc purchases were actually albums that I already owned on vinyl, but I bought them so that I could rediscover them on a new level. Now I hardly ever play my records unless I am hosting a graduation year playoff. I still own them, but they are doing something that records do unfortunately well: They are gathering dust.

I have lived long enough to see the very essence of my lifelong hobby redefined.

But records are only my hobby; my profession is teaching mathematics. I suppose I became interested in mathematics at about the same time I started collecting records. I was in seventh grade when the Russians launched Sputnik, thereby kicking off some interesting times for my mathematics teachers. My high school courses were taught out of paperback textbooks authored by the School Mathematics Study Group, code SMSG, whose approach, they predicted, would redefine the way we taught and learned mathematics.

That was the New Math, and it lasted long enough to develop a reputation bad enough to spawn the Back to Basics movement. "The Back to Basics Movement," they predicted, "will redefine the way we teach and learn mathematics."

Then suddenly we all got distracted by computers. Computers were doing well in the stock market, and were pretty much running most companies and the government, so there was a strong feeling that we should all find out how they work. What most people discovered was that they worked by *mathematics*, which was enough for most people, because it meant that mathematics teachers could henceforth be held responsible for explaining the remaining details to their children. Computers were installed in many schools, because, they predicted, "computers will redefine the way we teach and learn mathematics."

I was part of a three-man committee in 1978 that persuaded our Board of Trustees to sink 100 thousand dollars into a computer system that featured a Data General Nova 830 with 32K RAM, seven CRT terminals, and a teletype printer. Five years later we were back before them, hats in hand, pleading for another 100 thousand dollars to upgrade to a Hewlett-Packard 8000/30, plus ten more terminals and a sensible printer. Five years later we all but abandoned the HP and built a computer lab with shiny new Apple II's, at a cost of another 100 thousand dollars. Now we have an entire multimedia lab, fully stocked with Macintoshes, laser printers, scanners, CD-ROMs, and networking hardware, while our gleeful supplier has another 100 thousand of our school dollars. The amazing thing about this buying frenzy is that *each time* we pleaded our case with the Trustees, we assured them that *computers would redefine the way we taught and learned mathematics*. Incredibly, they fell for it every time.

But the sad truth of the matter was that we were still teaching and learning mathematics the way we had been doing it for decades. For all its marvelous capabilities, the computer was not changing the way that we taught and learned mathematics, and it was costing our school approximately 100 thousand dollars every five years to prove to ourselves that this was so.

Meanwhile, pocket scientific calculators had quietly appeared on the scene and had been welcomed in most mathematics classes. They *did* change a few things, but at such a mechanical level that people hardly noticed. Trig tables and log tables died a hasty and largely unmourned death, and now everyone had an equal chance at finding the purchase price of 4 CD's at $16.95 a piece, after a 10% discount and a 7.5% sales tax have been figured in. To most teachers, this was the sort of inconsequential drudgery that machines were *supposed* to do. Indeed, such was the inflexibility of the mathematics curriculum that these machines were welcomed precisely *because* they made so little difference in what we taught and learned. Significantly, the only place where they were at all controversial was at the elementary level, where people were nervous about children losing the ability to multiply and divide on paper. In any event, pocket calculators did not redefine the way we taught and learned mathematics.

By now you have probably figured out my little parable. SMSG and Back to Basics may have shifted things a little bit, but only in terms of decades, while the inflexibility of the mathematics curriculum must be measured on a geological scale. New points of emphasis come and go periodically, denting the curriculum monolith with the same approximate impact as that of the 8-track tape on the recording industry.

Scientific calculators were nice, and they even got into the classrooms, but they were the cassettes of our profession. Sure, everyone has one, but cassettes never replaced the records, which were still the way that serious people played their music, and scientific calculators never replaced factoring, which was what students *really* needed to know if they wanted to succeed in serious mathematics.

Computers took us to the laser disc stage, and failed to redefine the way we taught and learned mathematics for the same reasons that laser discs failed to redefine the recording industry. It had nothing to do with the capabilities of the technology, and everything to do with the mood of the market place. No machine can inspire a revolution if Hugh Hefner is the only one who owns one.

Of course, there is another chapter in this story. We are still writing that chapter as I write these words, but I firmly believe that you and I have lived long enough to see the compact disc of our profession: an instrument which is so perfectly suited to what we do, that it is in the process of redefining the way we teach and learn mathematics. We call this wonderful machine a graphing calculator, but it is in fact a computer, with ironically the same computing power in kilobytes as that first computer system we bought at my school fifteen years and 400 thousand dollars ago. With this machine my students can do far more than compute; they can conjecture, they can model, and they can make connections—the very things that I want to teach them to do. Moreover, they *own* this technology; they do not have to go over to a rich friend's home or to a special room at school to use it. Nor do I even have to tell them how or when to use it. Like the compact disc, it has become a part of their lives.

Oh sure, for a while I tried to treat this technology the way I treated the cassettes in the record store. I bought one and used it, but I never thought it would redefine my profession. After all, I had twenty years of my life invested in the math curriculum monolith, and I had become pretty successful at teaching the traditional courses in some embarrassingly traditional ways. But I was open to change, and I had read the *Standards*, so I began to chip away at my preconceptions of what and how I had to teach. The first thing I did was to let them use their graphing calculators all the time. The next thing I did was to start every class with a problem, which the students would talk out until a solution emerged that they could explain to each other. What I discovered, of course, was how useless my crisp set of lecture notes had been all these years. The students were discovering the results *without me*, and then *showing each other how to solve the problems*. That left me free to walk around and answer questions. Every so often I still tie things together or generalize, but for the most part I let the course evolve through what the students are doing, and I provide the direction by the problems I select. I'm still not sure what the heck I'm doing, but I do know this: There is more mathematics going on in my classroom these days than there ever has been before. Now there are more people doing it.

Was the graphing calculator responsible for transforming my entire approach to teaching? Well, yes and no. What the graphing calculator did was get me to question what I had been doing for twenty years. It also got me focused on how I would get the students using it, which in turn got me focused on student learning rather than my own teaching. Eventually, after sacrificing the first few sacred cows, I acquired a taste for sacred beef and the rest was easy. And believe me, that same thing is going on in mathematics classrooms all across the country, in a movement that is growing exponentially.

When I was young, all of my radical friends were in reform school. Today, all of my radical friends are in school reform. It is a crazy, free-wheeling time, not unlike the political scene in eastern Europe, and it's really pretty exciting—once you overcome the initial sensation of being totally lost in a Brave New World. Let's face it, though: For years we mathematicians were in a rather unique position in the world of academia, being smugly certain that we could *all* teach the *exact* same curriculum in the *exact* same way because we knew *exactly* what was best for

our students. The other subject area committees in the AP program fought among themselves all the time, while my colleagues and I on the Calculus committee nodded sagely in unison, reviewing yet another problem about a region being rotated about the x-axis. It was a happy, homogeneous, unrealistic world, totally incompatible with the spirit of creative discovery that had characterized the evolution of mathematics since the dawn of cognition.

Did you ever wonder what Newton would say if he could come back today and watch a traditional calculus class in action—if you could call it "action"? Do you suppose he would be flattered to see that, 300 years after his death, we were all teaching *his same results* to our students? I don't want to put words into the old Lion's mouth, but knowing that Newton once wrote to Robert Hooke, "If I have seen further it is by standing on the shoulder of giants," I dare say that he might pick up a graphing calculator, stare at it for several minutes with amazement, then say something like this: "The creators of this magic are giants. Is there nobody here who would stand on their shoulders?"

It is almost axiomatic in the academic world that the most creative minds belong to the mathematicians. Sadly, in the world of mathematics education that renowned creativity has been stifled for too long. To counter the trend, I tried a few years ago to release my own creative Muse in an evening of unbridled mathematical activity. I sat down with a fresh pad of paper and a pitcher of martinis, and before the evening was over and the martinis were gone I had produced 75 proofs of never-before-seen theorems. (Actually, I underachieved. It was an 80-proof gin.) I also created the perfect calculus problem, and here it is:

THE ALL-PURPOSE CALCULUS PROBLEM

A particle starts at rest and moves with velocity $v(t) = \int_1^t e^{-x^2}\,dx$ along a 10-foot ladder, which leans against a trough with a triangular cross-section two feet wide and one foot high.

Sand is flowing out of the trough at a constant rate of two cubic feet per hour, forming a conical pile in the middle of a sandbox which has been formed by cutting a square of side x from each corner of an 8" by 15" piece of cardboard and folding up the sides.

An observer watches the particle from a lighthouse one mile off shore, peering through a window shaped like a rectangle surmounted by a semicircle.

(a) How fast is the tip of the shadow moving?

(b) Find the volume of the solid generated when the trough is rotated about the y-axis.

(c) Justify your answer.

(d) Using the information found in parts (a), (b), and (c), sketch the curve on the pair of coordinate axes given.

Okay, maybe I went too light on the vermouth. But here's the sad part: Every teacher reading this article knew those problems by heart, because we've been teaching those same basic problems for years. That, indeed, is what calculus teachers have been doing for decades and decades. We have been teaching the math literate of tomorrow with the problems of yesterday, while explaining to them all the while that they will *need this mathematics in the future*. My friends, this is the stuff of which the Emperor's New Clothes are made! While we have been spinning golden oldies on the phonograph, perhaps more accurately the victrola, the world of Mathematical Reality has gone CD.

Today, thankfully, all of that is changing. To cite just one close-to-home example of that, the College Board has given final approval to the AP Committee's recommendation that the AP examinations be made graphing-calculator-active

beginning in 1995. We will also go to a new test format, splitting the multiple choice section into two 45-minute parts, one with 25 questions to be taken with *no* calculator, the other with 15 questions, some requiring a graphing calculator. The free response section will remain the same, 6 questions in 90 minutes, and will be designed with graphing calculators in mind. The *extent* to which calculators will be required will be slight at first, but may increase over time as emphases in the curriculum change.

For the next year or so, the Committee will be buried under test development details resulting from our decision. After that we will emerge to begin a careful evaluation of the calculus curriculum, from a to z. Or, if you like, from ε to δ. I recommend, however, that every teacher in the AP program get started without us, just as you started without us with graphing calculators. Don't worry about whether you are doing the same thing in your classroom as I am doing in mine; we have all worried about that for too long. Just worry about whether your students are learning calculus and enjoying it. If that's the *only* thing that our classrooms have in common, then our students are still much better off than when we had *everything else* in common at the expense of that.

In the next few years, the vibrations from all these education reform movements will reach a crescendo, and from it all will emerge a new paradigm for teaching and learning. I have seen enough to realize that it will not be confined only to mathematics; nonetheless, it is apparent to any observer that mathematics is leading the way. If Newton does come back to visit us, I hope he waits a few years so that all of this will have had a chance to develop. Then, when he asks to see the Calculus for the Twenty-first Century, we can show him something which Newton would truly appreciate: an entire Renaissance Curriculum.

And I'll bet we get to show it to him on a CD—hooked up to our graphing calculator.

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An analyst, surname of Nero
Is my mathematical hero.
Says he, "When in doubt,
I always start out,
'Given $\epsilon > 0 \ldots$'"

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